



RESEARCH MEMORANDUM

DETERMINATION OF BLADE-TO-COOLANT HEAT-TRANSFER
COEFFICIENTS ON A FORCED-CONVECTION,
WATER-COOLED, SINGLE-STAGE TURBINE

By John C. Freche and Eugene F. Schum

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

WASHINGTON

July 13, 1951

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SUMMARY

Blade-to-coolant convection heat-transfer coefficients were obtained on a forced-convection, water-cooled, single-stage turbine over a range of turbine speeds, coolant flow rates, and inlet-gas temperatures. The convection coefficients were determined over a large laminar-flow range and over a portion of the transition range between laminar and turbulent flow. The range of Graetz numbers investigated was from 230 to 2400.

Both natural- and forced-convection heat transfer were anticipated in this turbine. Natural-convection heat transfer, expressed nondimensionally by a relation between the Nusselt and the Grashof numbers, was negligible for this turbine over the Grashof number range investigated from 3×10^9 to 4×10^{12} . Blade-to-coolant convection coefficients were correlated by the general relation for forced-convection heat transfer for laminar flow that expresses Nusselt number as a function of Graetz number. When a characteristic dimension in the Graetz number equivalent to the length of a radial coolant passage in the blade was used, the turbine data agreed closely with a curve representing the data obtained by other investigators for laminar flow of heated liquids through stationary tubes.

When blade-to-coolant heat-transfer coefficients from stationary-tube heated-liquid data and theoretical gas-to-blade coefficients were used to calculate midspan average blade temperatures, the maximum deviation of calculated values from experimental data was 19°F at an experimental blade temperature of 165°F .

INTRODUCTION

In order to design cooled turbine blades, it is necessary to calculate blade temperatures for the desired design operating conditions. Theoretical equations have been derived for the calculation of blade temperatures; however, to calculate these blade temperatures, gas-to-blade and blade-to-coolant heat-transfer coefficients are required.

An investigation of blade-to-coolant heat transfer for a forced-convection, water-cooled, single-stage turbine is described herein.

A theoretical relation for surface-to-coolant convective heat transfer for fully-developed laminar flow of fluids through stationary tubes with an unheated entrance length is presented in reference 1. Experimental investigations of heat transfer with laminar flow of heated liquids through stationary tubes have been conducted by several investigators (reference 1). In these investigations, correlation of the data for the laminar flow range is achieved by a general relation that expresses Nusselt number as a function of Graetz number.

Blade-to-coolant convection heat-transfer data were also obtained as part of an earlier investigation of a forced-convection, water-cooled, single-stage turbine (reference 2) over a limited range of turbine speeds and inlet-gas temperatures; the data were correlated in a similar manner to that for stationary tubes. In reference 2, a satisfactory agreement was not achieved between the correlated turbine data and the stationary-tube investigations (reference 1) over the laminar flow range. The investigation of reference 2 gave indications that blade-to-coolant natural-convection heat transfer was also encountered in the forced-convection, water-cooled, single-stage turbine. The effect of natural convection upon blade-to-coolant heat transfer could not be evaluated because of insufficient data.

The investigation of blade-to-coolant heat transfer in cooled turbine blades was continued on a forced-convection, water-cooled turbine at the NACA Lewis laboratory over an extended range of turbine speeds and inlet-gas temperatures. The investigation was conducted to determine: (a) the magnitude of the natural-convection effect upon the blade-to-coolant heat-transfer coefficient, (b) if better agreement between turbine blade-to-coolant heat-transfer data and stationary-tube data could be achieved, and (c) if blade temperatures could be accurately calculated using stationary-tube blade-to-coolant coefficients obtained from the correlation method shown in reference 1 and a theoretical relation for gas-to-blade coefficients determined for this turbine (references 3 and 4). Blade-to-coolant convection heat-transfer coefficients presented herein were obtained over a range of turbine speeds from 2000 to 14,500 rpm and over a range of gas temperatures from 400° to 1300° F. Coolant flow rates from 0.7 to 7 gallons per minute, equivalent to a range of mass velocities from 1565 to 15,650 pounds per minute per square foot, were employed.

APPARATUS

Turbine

The forced-convection, water-cooled, single-stage aluminum turbine operated in this investigation is fully described in reference 2. A cutaway section of the turbine blade and a cross section of the turbine installation are shown in figure 1. Blade construction details and the coolant flow path through the blade are shown in figure 1(a). The turbine has 50 impulse blades of constant cross section and with no twist. The turbine rotor has a tip diameter of 12.06 inches, a blade chord of 0.744 inch, and a blade span of 1.15 inches. Coolant passages near the leading and trailing edges of the blade have a 0.062-inch diameter and coolant passages near the blade center have a 0.099-inch diameter. The crossover passage near the blade tip connecting the radial coolant passages is 0.062 inch in diameter. Figure 1(b) shows the turbine installation and the coolant path through the entire turbine.

Instrumentation

The planes of instrumentation through the rig are shown in figure 1(b). Blade temperatures were obtained by means of a rotating thermocouple pick-up consisting of a slip-ring and brush system. Locations of four of the six thermocouples on the rotor blades are indicated in figure 1(a). At the pitch diameter, thermocouples were located at the leading and trailing edges, and at points on the pressure and suction surfaces approximately midway between the leading and trailing edges. In addition, a thermocouple was located at the center of the pressure surface at the blade tip and another at the center of the suction surface at the blade root. A thermocouple on the water-outlet side of the baffle plate indicated the water temperature immediately prior to discharge from the rotor. A stationary thermocouple located in the coolant supply pipe was used to measure the temperature of the coolant entering the turbine rotor. Total and static pressure, gas temperature, turbine speed, and coolant flow rate were measured in the manner described in reference 2.

METHODS OF CALCULATION

Evaluation of Blade-to-Coolant Heat-Transfer Coefficients

A rigorous and an approximate calculation procedure were used in determining blade-to-coolant heat-transfer coefficients in this investigation.

Rigorous calculation procedure. - For this turbine, the total heat absorbed by the coolant consists of

(a) The heat transferred to the coolant by convection in the blade coolant passages, Q_B (See fig. 2, section 2) (All symbols are defined in appendix A.)

(b) The heat transferred to the coolant from the section of disk rim between the blades (exposed peripheral disk rim area), Q_h

(c) The heat transferred by radial conduction from blade to disk rim, Q_c

Heat transferred by the turbine shaft was considered negligible. The total heat transferred to the coolant was determined from the coolant weight flow and the coolant temperature rise by the expression

$$Q_T = wc_p \Delta T_l \quad (1)$$

The coolant temperature rise was evaluated by the rotor-coolant inlet and outlet thermocouples. Turbine instrumentation did not permit independent determination of the heat transferred by convection to the coolant in the blade coolant passages (fig. 2, section 2). The heat flow of items (b) and (c) can be calculated, however. Convective heat transfer in the blade coolant passages can thus be isolated and a convective blade-to-coolant heat-transfer coefficient can be obtained. The convective heat transfer to the coolant in the blade is obtained from the expression

$$Q_B = Q_T - (Q_h + Q_c) \quad (2)$$

Heat transferred to the coolant from the section of disk rim exposed to the hot gases (peripheral section between blades) is determined from the equation

$$Q_h = H_o S_h (T_{g,e} - T_h) \quad (3)$$

An average gas-to-blade heat-transfer coefficient evaluated in the manner described in references 2 and 4 was used in equation (3). The effective gas temperature $T_{g,e}$ was obtained from equation (3) of reference 2. The thermocouple at the blade base provided the temperature T_h used for the disk rim. The expression used to determine the heat transferred by radial conduction from the blade to the disk rim is

$$Q_c = -k_B A_2 \left(\frac{dT_B}{dx_2} \right)_{x_2 = L_f} \quad (4)$$

The radial temperature gradient at the blade root $(dT_B/dx_2)_{x_2=L_f}$ in equation (4) is obtained by the method described in appendix B. Average blade-to-coolant convective coefficients were then calculated by the equation

$$H_1 = \frac{Q_B}{S_1 (T_{B,av} - T_{l,av})} \quad (5)$$

The integrated average blade temperature was obtained from a plot of midspan blade temperature against blade periphery. The coolant temperature was taken as the average of the rotor-inlet and -discharge coolant temperatures.

Approximate calculation procedure. - Since the determination of the heat-transfer coefficient by accounting for heat transferred through radial conduction is a tedious and time-consuming process, the bulk of the data points were calculated using an approximate calculation method. Heat-transfer coefficients were calculated from equations (1) to (3) and (5) with the omission from equation (2) of the heat transferred by radial conduction Q_c . These coefficients were used in calculating Nusselt numbers, the dimensionless heat-transfer parameters employed in the data correlations. From a limited number of data points, a relation was obtained by plotting Nusselt numbers incorporating coefficients calculated by a method that neglects the radial conduction against Nusselt numbers incorporating coefficients calculated by the rigorous procedure previously described. The relation was then applied to approximately 100 data points thereby converting all the data to a basis which accounts for radial conduction.

Correlation of Blade-to-Coolant Heat-Transfer Coefficients

Natural-convection heat-transfer correlation. - Blade-to-coolant convective heat-transfer coefficients obtained over a range of Grashof numbers for several constant Graetz numbers were correlated on the basis of an expression for natural-convection heat transfer

$$Nu = f(Gr) \quad (6)$$

Fluid properties were based on the average coolant temperature. Prandtl number was omitted from the correlation equation as in another investigation (reference 1) of combined natural- and forced-convection heat transfer in a stationary tube. The characteristic dimension L_n in the Grashof number was considered to be the length of the water column or average radial coolant passage length (fig. 2) in accordance with the theory explained in reference 5. The expression for the Grashof number for a rotating coolant passage requires a significant change from the

expression for Grashof number for a stationary tube; the centrifugal forces acting on the fluid far exceed the force due to gravity. The acceleration term $\omega^2 r$ is therefore used in the expression for the Grashof number. The radius r is considered to be the radius to the centroid of a water column with a length equivalent to the average coolant-passage length. Rather than use the customary length L_n , a characteristic dimension D equal to four times the sum of the cross-sectional areas of the coolant passages divided by the sum of the passage perimeters was used in the Nusselt number. The characteristic dimension D was employed to maintain a common basis for both natural- and forced-convection correlations. The instrumentation did not permit determination of the coolant flow rate through each passage; therefore, such an average hydraulic diameter rather than the individual passage diameters was used.

Forced-convection heat-transfer correlation. - Blade-to-coolant convective heat-transfer coefficients were also correlated by a general relation expressing forced-convection heat transfer with laminar flow of heated liquids in tubes:

$$Nu = f(Gz) \quad (7)$$

The data were correlated by this formula because coolant flows were predominantly laminar and extended somewhat into the transition range between laminar and turbulent flow. The Reynolds number was calculated for each coolant flow rate to determine whether the data were above or below the critical Reynolds number of 2300. The length of the radial coolant passage in the blade was used as the characteristic dimension L_f in the Graetz number. Although heat is transferred to the coolant along the coolant passage as it passes through the disk rim and through the blade, insufficient temperature measurements are available to determine exactly the heat flow conditions in each of these sections. No thorough investigations as cited in reference 6 have been made of heat transfer in stationary tubes heated over the entire length beginning at the intake cross section. Because most of the heat is transferred to the coolant over the passage length within the blade, this portion of the coolant passage was considered comparable to the heated length of a stationary tube and consequently used as the characteristic dimension in the Graetz number. In this correlation, fluid properties were based upon the average coolant temperature, and the characteristic dimension D used in the Nusselt number was identical to that described previously.

Calculation of Average Blade Temperatures

In order to determine the accuracy with which blade temperatures can be calculated for this turbine using available heat-transfer

coefficients, midspan average blade temperatures were calculated. Temperature distribution equations derived in reference 7 and modified in appendix B were used. The expression (equation (B2) of appendix B) used to obtain the blade temperature is

$$T_{B,2} = T_{B,p} + C_3 e^{\alpha x_2} + C_4 e^{-\alpha x_2} \quad (8)$$

where the constants and exponents are evaluated in the manner stated in appendix B. Blade-to-coolant and gas-to-blade heat-transfer coefficients are embodied in this expression. Average midspan blade temperatures were calculated using the blade-to-coolant coefficients obtained from the stationary-tube data curve for heated liquids and the theoretical gas-to-blade coefficients for this turbine obtained in reference 4 by the method of reference 3.

PROCEDURE

The turbine was operated to obtain blade-to-coolant heat-transfer coefficients over a wide range of coolant flow, turbine speed, and inlet-gas temperatures. Over 125 runs were made to establish the blade-to-coolant heat-transfer correlation curve. In order to determine the effect of natural convection on blade-to-coolant heat transfer, the turbine was operated over a wide range of Grashof numbers at constant coolant-flow Graetz numbers. The effect of forced convection on blade-to-coolant heat transfer was determined by turbine operation over a wide range of coolant-flow Graetz numbers. The scope of turbine operating conditions is given in table I. Variation in Grashof number was achieved by changing turbine speed and inlet-gas temperature at a constant coolant flow rate, thereby altering the acceleration term $\omega^2 r$ and the blade-to-coolant temperature difference $(T_{B,av} - T_{l,av})$, the factors which most affect the magnitude of the Grashof number. Variations in Graetz number were obtained by altering the coolant flow. Turbine speed was adjusted by means of a water-brake, and the coolant flow was independently regulated to any desired value. In order to insure longer turbine life, operation was maintained at reasonable rotor stress levels by not exceeding 14,500 rpm. This speed represents slightly more than one-half of the design centrifugal loading. Inlet-gas temperatures ranged from 400° to 1300° F. The coolant flow range extended from 0.7 to 7 gallons per minute, the maximum flow obtainable with this turbine installation.

RESULTS AND DISCUSSION

Effect of Radial Conduction on Blade-to-Coolant Heat Transfer

A comparison of representative data points of Nusselt numbers obtained by incorporating coefficients calculated by considering radial conduction and Nusselt numbers incorporating coefficients calculated omitting radial conduction is shown in figure 3. When radial conduction from the blade to the disk rim was considered, the Nusselt numbers were decreased a minimum of 14 percent and a maximum of 31 percent over the range considered. Figure 3 also indicates, that for this turbine, the effect of conduction is appreciable. A short blade span, large blade cross-sectional area, and high conductivity of the blade material ($105 \text{ Btu}/(\text{hr})(\text{ft})(^\circ\text{F})$) facilitate conduction of heat through the blade to the disk rim. Radial blade-temperature-distribution curves for longer blades of both high- and low-conductivity materials are presented in reference 7. Analysis of these curves indicates that with increased thermal conductivity, the influence of rim cooling (radial conduction) is more pronounced. For longer blades of low-conductivity, heat-resistant alloys, the effect of conduction is probably negligible.

Correlation by Expression for Natural Convection

Although forced-convection circulation of the rotor coolant is employed in this turbine, the high rotational speeds are conducive to natural-convection circulation in the coolant passages as well. Consequently, the blade-to-coolant heat-transfer rate should be affected. The data obtained at several constant Graetz numbers, 460, 612, 1275, and 2230, were correlated by an expression for natural-convection heat transfer, and these correlations are shown in figure 4. No increase in Nusselt number is apparent for Grashof numbers from 3×10^9 to 4×10^{12} , indicating that natural convection has a negligible effect upon blade-to-coolant heat transfer for this turbine. The curves indicate that the Nusselt number increases for increasing values of Graetz number. Since Graetz number is a function of coolant weight flow and Grashof number is primarily a function of turbine speed, the data further indicate that coolant flow and not speed govern blade-to-coolant heat transfer for this turbine. Consideration of coolant velocity profiles in the coolant passages provides a possible explanation for the apparently negligible effect of natural convection. For natural-convection flow a certain velocity profile exists (reference 8). If natural-convection flow exists in the blade leading edge passages, the relatively cool core of fluid is directed radially outward. The heated fluid in the boundary layer adhering to the passage wall is directed radially inward. When the velocity component of forced-convection flow is added vectorially to the velocity component of the heated fluid in the boundary layer (flow radially inward) the resultant velocity is

reduced. Because the boundary-layer velocity governs the rate of heat transfer, a decrease in convective heat transfer occurs in the leading-edge passages. A similar analysis for the trailing-edge passages in which the forced-convection flow is radially inward indicates an increase in the boundary-layer velocity component, and a corresponding increase in heat transfer. The net effect on the average blade-to-coolant heat-transfer coefficient is therefore probably negligible.

Correlation by Expression for Forced Convection

Experimental data of other investigators, shown in reference 1 for the laminar flow of heated and cooled liquids through stationary tubes, are presented in figure 5. These data were obtained from tubes containing adiabatic calming sections preceding the heat-transfer length. It is inferred in reference 1 that the calming sections were not of sufficient length to have fully developed flow over the entire heat-transfer length. There is little available information, however, concerning the effect of length of calming section on heat transfer for laminar flow in tubes. A theoretical relation for the fully developed laminar flow of fluids through stationary tubes with an unheated entrance length is given in reference 1 and is also presented in figure 5. The theoretical relation is displaced from the heated data. A mean curve has been drawn through the heated stationary-tube data in figure 5 to facilitate comparison with turbine data reported herein.

All blade-to-coolant heat-transfer data obtained in this investigation are shown in figure 6. The curve representing data for the laminar flow of heated liquids through stationary tubes (fig. 5) passes through the turbine data (fig. 6). Maximum deviation of turbine data from the stationary-tube curve was 17 percent for a Graetz number range of 600 to 2400, a range for normal operation of this turbine under current operating conditions. At the lowest Graetz number of 230, the maximum deviation of turbine data from the stationary-tube curve is 35 percent. Turbine data, with the exception of several low Graetz number points, fall well within the scatter of the stationary-tube data as indicated in figure 6. The largest deviation generally occurs below Graetz numbers of 500. A similar tendency is evidenced by the stationary-tube data (fig. 5), which also show the largest spread at low Graetz numbers.

Because Nusselt number is a function of Graetz number, the correlation is dependent upon the characteristic dimension used in obtaining the Graetz number. Consequently, the degree of agreement obtained between turbine data and stationary-tube data is also influenced by the characteristic length used in the turbine-data correlations. As cited previously, turbine data were correlated using what appears to be a reasonable characteristic length; however, insufficient information is available to permit selection of this term with certainty.

An earlier investigation described in reference 2 shows blade-to-coolant heat-transfer data obtained with this turbine neglecting radial conduction. Since the investigation reported in reference 2, a more satisfactory technique of installing blade thermocouples was evolved. Only data obtained with the new instrumentation are reported herein.

Calculated Average Midspan Blade Temperatures

The usefulness of a blade-to-coolant heat-transfer correlation obtained with a turbine is best indicated by the accuracy with which blade temperatures can be calculated employing this correlation. Because experimental blade-to-coolant heat-transfer coefficients agree well with stationary-tube data and may not be available for a particular blade-coolant passage configuration, the desirability of using existing stationary-tube coefficients is obvious. Use of a theoretically calculated gas-to-blade heat-transfer correlation is similarly desirable. Figure 7 presents a comparison of experimental and calculated average midspan blade temperatures obtained by using blade-to-coolant coefficients from the stationary-tube curve shown in figures 5 and 6 and gas-to-blade coefficients calculated for this turbine from boundary-layer theory (references 3 and 4). The maximum deviation of the calculated values from the experimental data was 19°F at an experimental blade temperature of 165°F . These results indicate that theoretical gas-to-blade and stationary-tube blade-to-coolant heat-transfer coefficients may be used to calculate average midspan blade temperatures for this turbine under imposed gas and coolant conditions.

It should be emphasized that the turbine blade temperatures calculated are for a material of high conductivity and are an average of individual midspan blade temperatures. For blades of low conductivity and for locations at some distance from the coolant passage, the calculation of individual blade temperatures will require use of additional equations derived in reference 7.

SUMMARY OF RESULTS

The following results were obtained from the investigation of blade-to-coolant heat transfer conducted on a forced-convection, water-cooled aluminum turbine:

1. For this turbine, the natural-convection effect on the blade-to-coolant heat-transfer coefficient was negligible over the range of turbine speeds and coolant flow rates investigated.

2. Blade-to-coolant convection coefficients for this turbine were correlated by the general relation for forced-convection heat transfer with laminar flow that expresses the Nusselt number as a function of the Graetz number.

3. When a characteristic dimension in the Graetz number equivalent to the length of a radial coolant passage in the blade was used, the turbine data agreed closely with a curve representing the data for laminar flow of heated liquids through stationary tubes.

4. When blade-to-coolant heat-transfer coefficients from stationary-tube heated-liquid data and theoretical gas-to-blade coefficients are used to calculate midspan average blade temperatures, the maximum deviation of calculated values from experimental data was 19° F at an experimental blade temperature of 165° F.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio.

APPENDIX A

SYMBOLS

The following symbols are used in this report:

a	defined by $(H_o l_o / k_B A_1)^{1/2}$, ft ⁻¹
A	area normal to radial heat conduction path in blade, sq ft
c _p	specific heat of coolant at constant pressure, Btu/(lb)(°F)
C ₁ to C ₆	constants
D	hydraulic diameter of coolant passages, ft
Gz	Graetz number $w c_p / k L_f$
Gr	Grashof number $\frac{L_n^3 \rho^2 \beta \omega^2 r (T_{B,av} - T_{l,av})}{\mu^2}$
H	heat-transfer coefficient, Btu/(°F)(sq ft)(sec)
k	thermal conductivity of coolant unless otherwise indicated by subscripts, Btu/(°F)(ft)(sec)
l	perimeter, ft
L	length of coolant passage in Graetz and Grashof numbers (fig. 2), ft
Nu	Nusselt number, $\frac{H_1 D}{k}$
Q	heat flow, Btu/sec
r	radius to centroid of water column, ft
S	surface area, sq ft
T	temperature, °F
T _{B,p}	defined by $\frac{H_o l_o T_{g,e} + H_1 l_1 T_l}{H_o l_o + H_1 l_1}, \quad ^\circ F$

w	coolant weight flow, lb/sec
x	distance along blade span defined in figure 2
α	defined by $\left(\frac{H_o l_o + H_i l_i}{k_B A_2} \right)^{1/2}$, ft ⁻¹
β	coefficient of thermal expansion of coolant, °R ⁻¹
Δ	prefix to indicate change
μ	absolute viscosity of coolant, slugs/(ft)(sec) or (lb)(sec)/sq ft
φ	defined by $(H_i l_i / k_B A_3)^{1/2}$, ft ⁻¹
ρ	mass density of coolant, slugs/cu ft
ω	angular velocity, radians/sec

Subscripts:

1,2,3	rotor sections illustrated in figure 2
av	average
B	blade
c	conduction
e	effective
f	forced convection correlation
g	gas
h	disk rim (exposed peripheral section between blades)
i	inside blade surface
l	coolant
n	natural convection correlation
o	outside blade surface
p	prevalent
T	total

APPENDIX B

EVALUATION OF TEMPERATURE GRADIENT AT BLADE ROOT

FOR USE IN EQUATION (4)

In order to evaluate the temperature gradient at the blade root, the turbine rotor was divided in three sections, as shown in figure 2. Section 1 consists of the blade tip that is heated by the hot gases and cooled only by blade thermal conduction. Section 2 consists of the portion of the blade that is heated by the hot gases and cooled by convection as well as by conduction. Section 3 consists of a portion of the disk rim cooled by convection and conduction. For each of these sections, radial-temperature-distribution equations were obtained from heat balances of incremental segments in each section as described in appendix B of reference 7.

The temperature gradient at the blade root is dependent upon the following temperature equations for the three sections: For section 1,

$$T_{B,1} = T_{g,e} + C_1 e^{ax_1} + C_2 e^{-ax_1} \quad (B1)$$

in which

$$a = \left(\frac{H_o l_o}{k_B A_1} \right)^{1/2}$$

The hyperbolic form of this equation is derived in reference 7. The equation for section 2, derived in reference 7, is

$$T_{B,2} = T_{B,p} + C_3 e^{\alpha x_2} + C_4 e^{-\alpha x_2} \quad (B2)$$

where $T_{B,p}$ and α are defined as follows:

$$T_{B,p} = \frac{H_o l_o T_{g,e} + H_i l_i T_i}{H_o l_o + H_i l_i}$$

and

$$\alpha = \left(\frac{H_o l_o + H_i l_i}{k_B A_2} \right)^{1/2}$$

The equation for the rim section of the rotor (section 3) is

$$T_{B,3} = T_l + C_5 e^{\varphi x_3} + C_6 e^{-\varphi x_3} \quad (B3)$$

where

$$\varphi = \left(\frac{H_1 l_1}{k_B A_3} \right)^{1/2}$$

Equation (B3) was derived from equation (B2), by letting the gas-to-blade coefficient H_0 equal zero. It is assumed that hot gases do not impinge upon the sides of the turbine rim section shown in figure 2. Clearances between the rotor and the nozzle-box installation are small enough to lend validity to this assumption. Consequently, the gas-to-blade coefficient for the sides of section 3 is equal to zero.

The temperature gradient $(dT_B/dx_2)_{x_2=L_f}$ for use in equation (4), is obtained by taking the derivative of equation (B2) and results in the equation

$$\left(\frac{dT_B}{dx_2} \right) = \alpha \left(C_3 e^{\alpha x_2} - C_4 e^{-\alpha x_2} \right) \quad (B4)$$

in which C_3 and C_4 are evaluated from equations (B1) to (B3) by the algebraic method of reference 7. The method of solution was simplified somewhat by assuming that the temperature at the inner diameter of the rim was equal to the average coolant temperature. Since the entire rim is in contact with a free flowing annulus of coolant subject to the influence of turbine rotational velocity, such an assumption is probably justified. It was necessary to obtain the values of C_3 and C_4 by a trial-and-error solution employing numerical values for the geometry factors listed in table II. A value for the blade-to-coolant coefficient H_1 was first assumed under the imposed gas and coolant conditions and values for C_3 and C_4 were obtained. These values were substituted in equation (B4) to determine an approximate value of the temperature gradient $(dT_B/dx_2)_{x_2=L_f}$. The value of $(dT_B/dx_2)_{x_2=L_f}$ was in turn substituted in equation (4) to obtain the approximate value of conduction heat flow Q_c from blade to disk. By use of equations (2) and (5), a value of H_1 was determined and compared with the assumed value. This procedure was repeated until absolute agreement with assumed values of the blade-to-coolant coefficient H_1 rendered further trials unnecessary.

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TABLE I - RANGE OF OPERATING CONDITIONS FOR BLADE-TO-COOLANT HEAT-TRANSFER INVESTIGATION



Objective	Coolant flow (gal/min)	Turbine speed (rpm)	Turbine-inlet gas temperature (°F)	Temperature difference: blade-coolant (°F)	Graetz number	Grashof number
Natural-convection effect and forced-convection effect	1.4	8800 - 13,500	400 - 1300	34 - 149	460	8.2 - 380X10 ¹⁰
	2.0	2000 - 14,500	400 - 800	17 - 78	612	3.2 - 600X10 ⁹
	4.0	2100 - 12,100	400 - 1200	21 - 106	1275	7.3 - 670X10 ⁹
	7.0	2200 - 14,500	400 - 1200	20 - 100	2230	3.6 - 440X10 ⁹
Forced-convection effect	0.7 - 7.0	2000 - 14,500	400 - 1300	16 - 150	230 - 2400	-----

TABLE II - DIMENSIONS OF TURBINE-BLADE GEOMETRY FACTORS



Passage length ^a used in G_z , L_f , (ft)	0.088
Passage length ^a used in G_r , L_p , (ft)	0.131
Hydraulic diameter of coolant passages, D , (ft)	0.00709
Blade cross-sectional area ^a , A_1 , (sq ft/blade)	0.001128
Blade cross-sectional area ^a , A_2 , (sq ft/blade)	0.000980
Disk rim cross-sectional area ^a , A_3 , (sq ft/blade)	0.004040
Coolant passage surface area inside blade, S_1 , (sq ft/blade)	0.00799

^aSee figure 2.

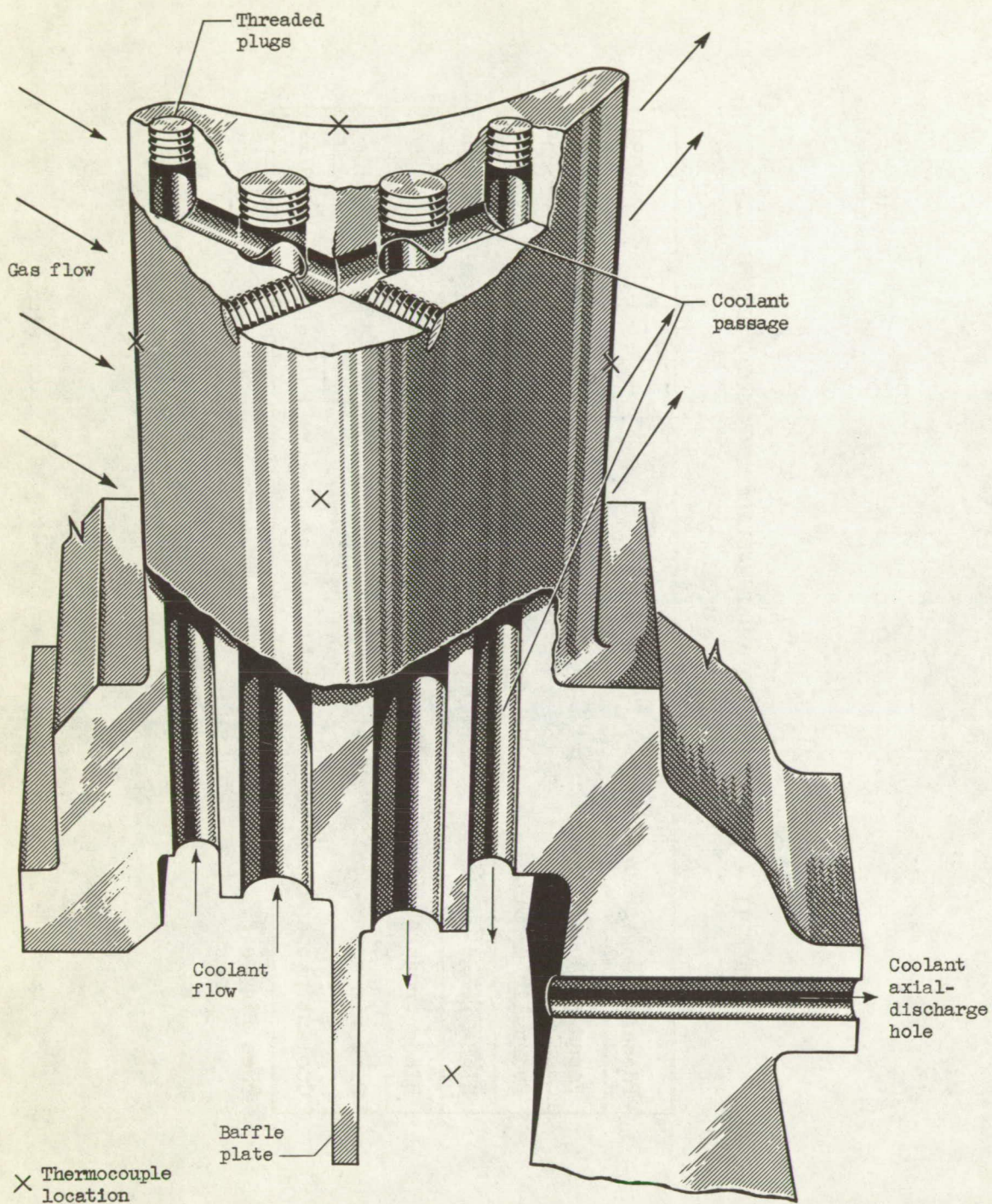
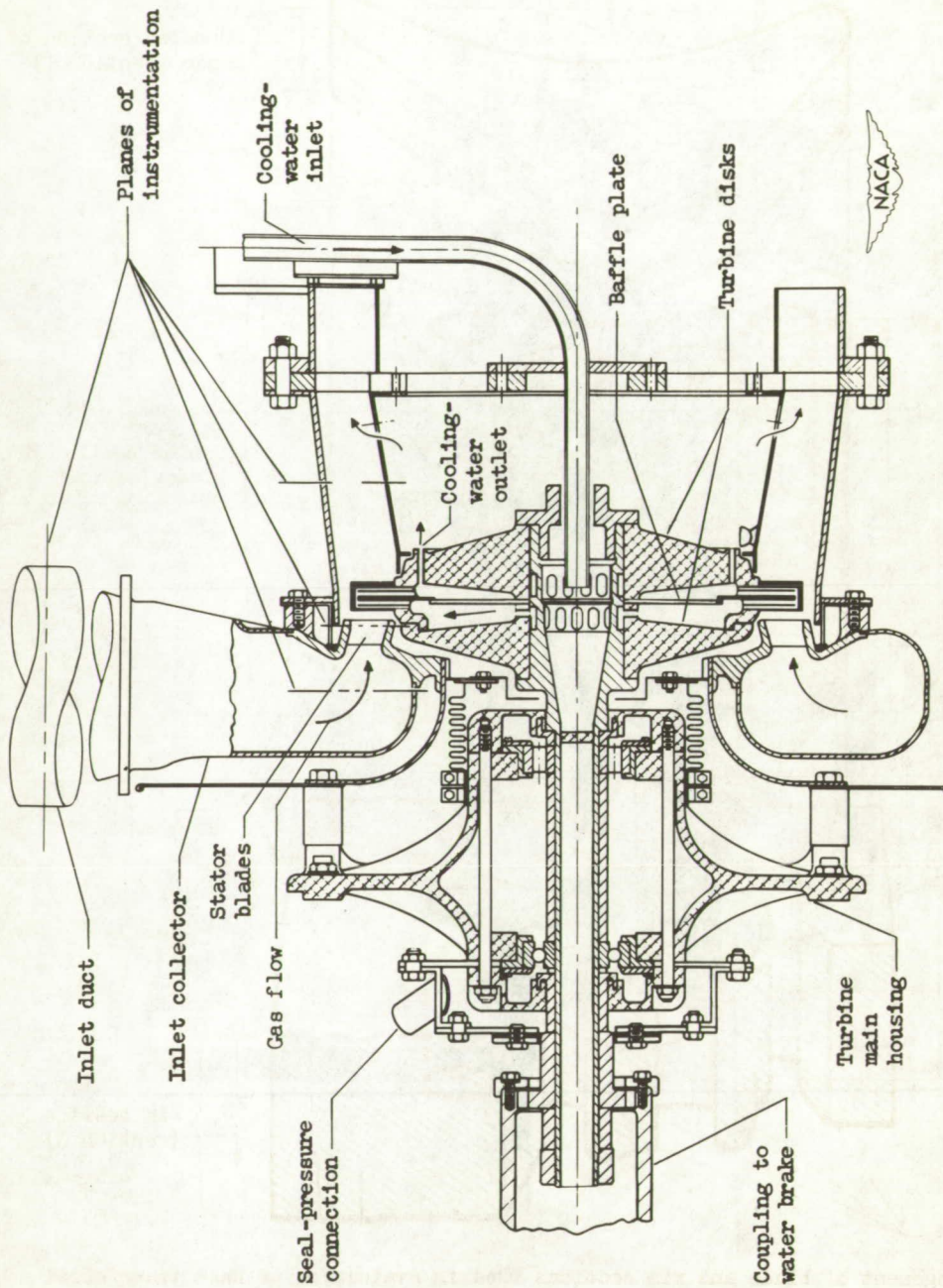


Figure 1. - Forced-convection water-cooled aluminum turbine.



(b) Cross section of installation.

Figure 1. - Concluded. Forced-convection water-cooled aluminum turbine.

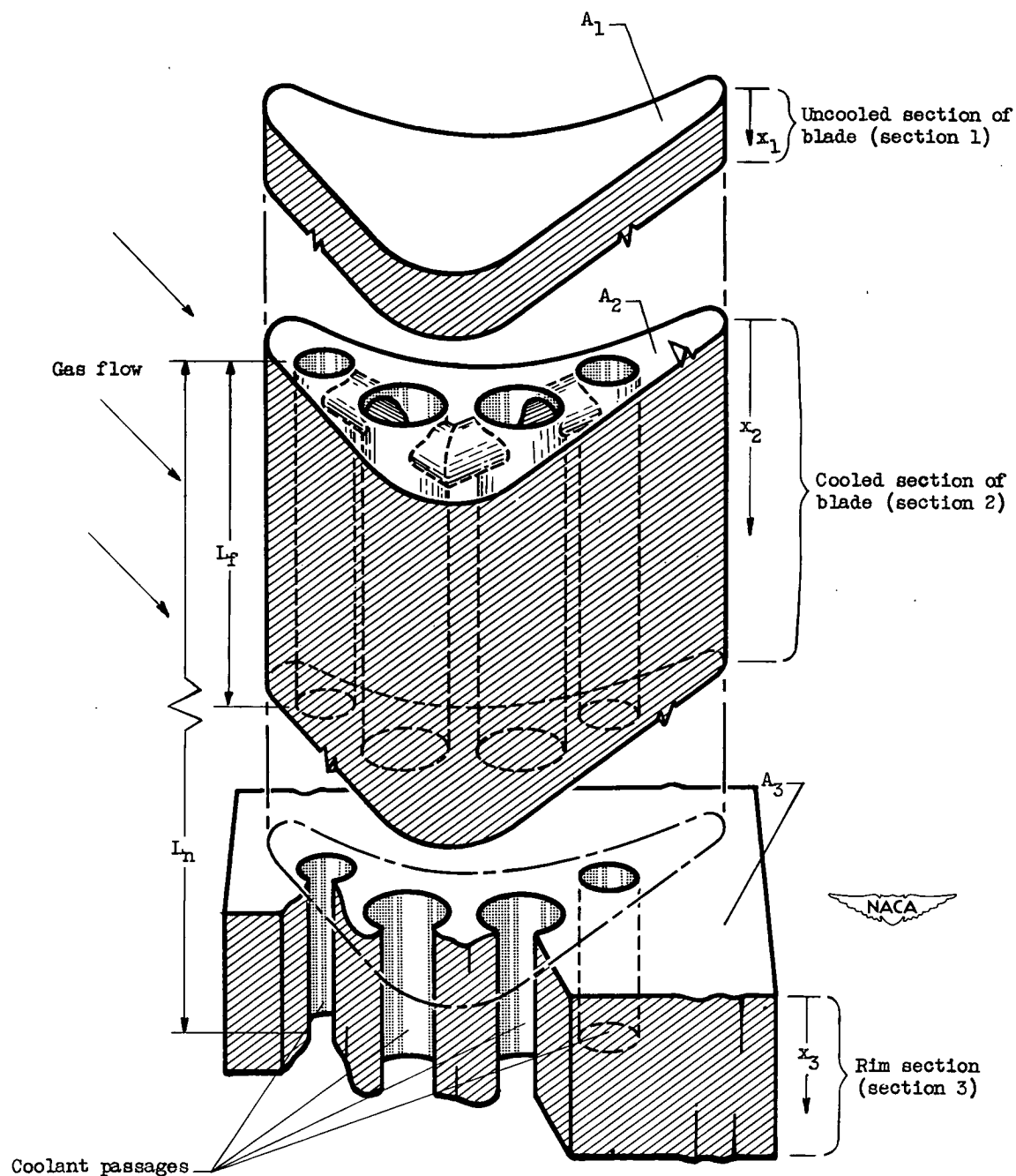


Figure 2. - Arrangement of blade and rim sections used in evaluation of heat transferred by thermal conduction from blade to rim.

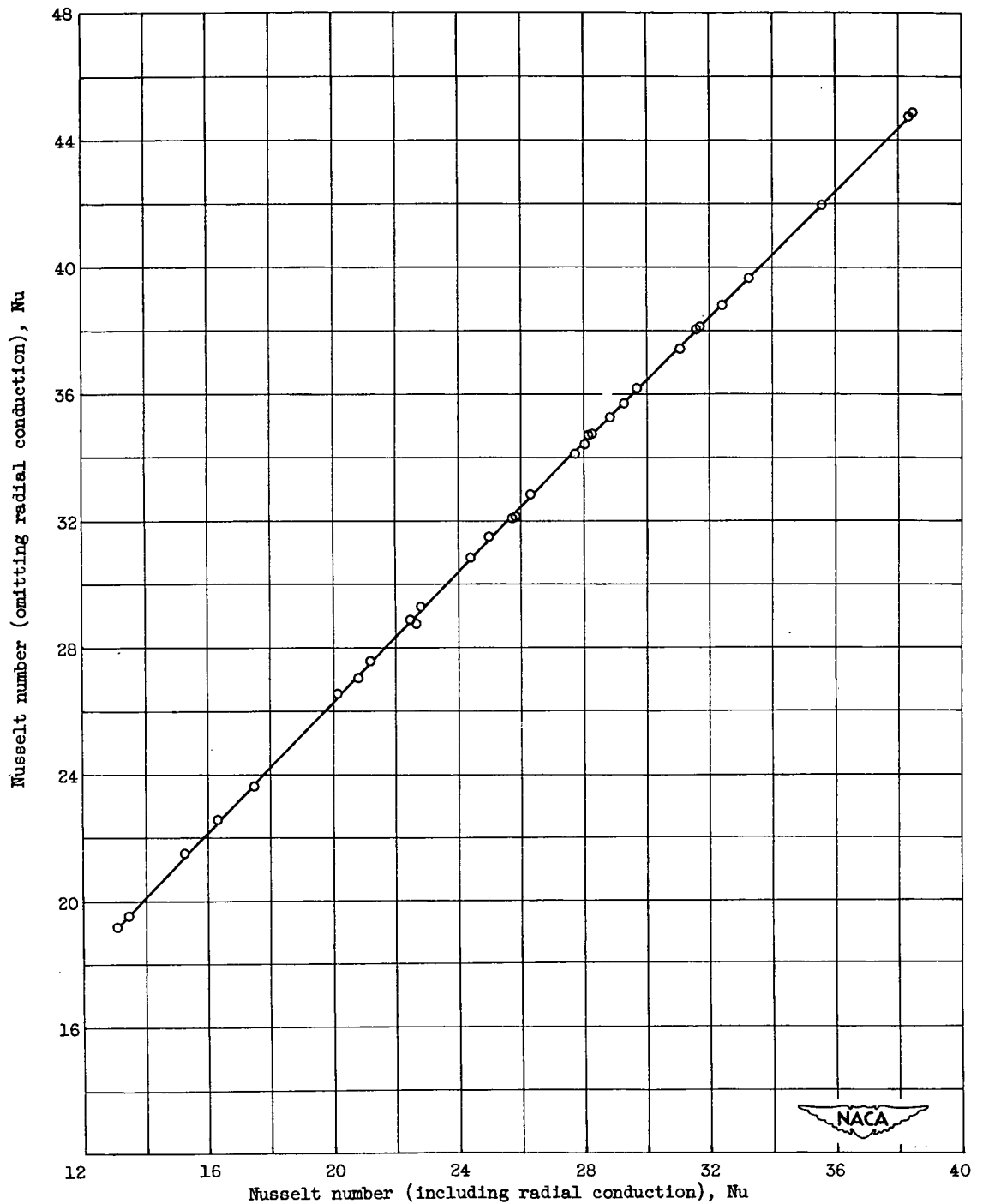


Figure 3. - Relation between blade-to-coolant Nusselt numbers for forced-convection water-cooled turbine showing effect of radial conduction. Fluid properties based on average coolant temperature.

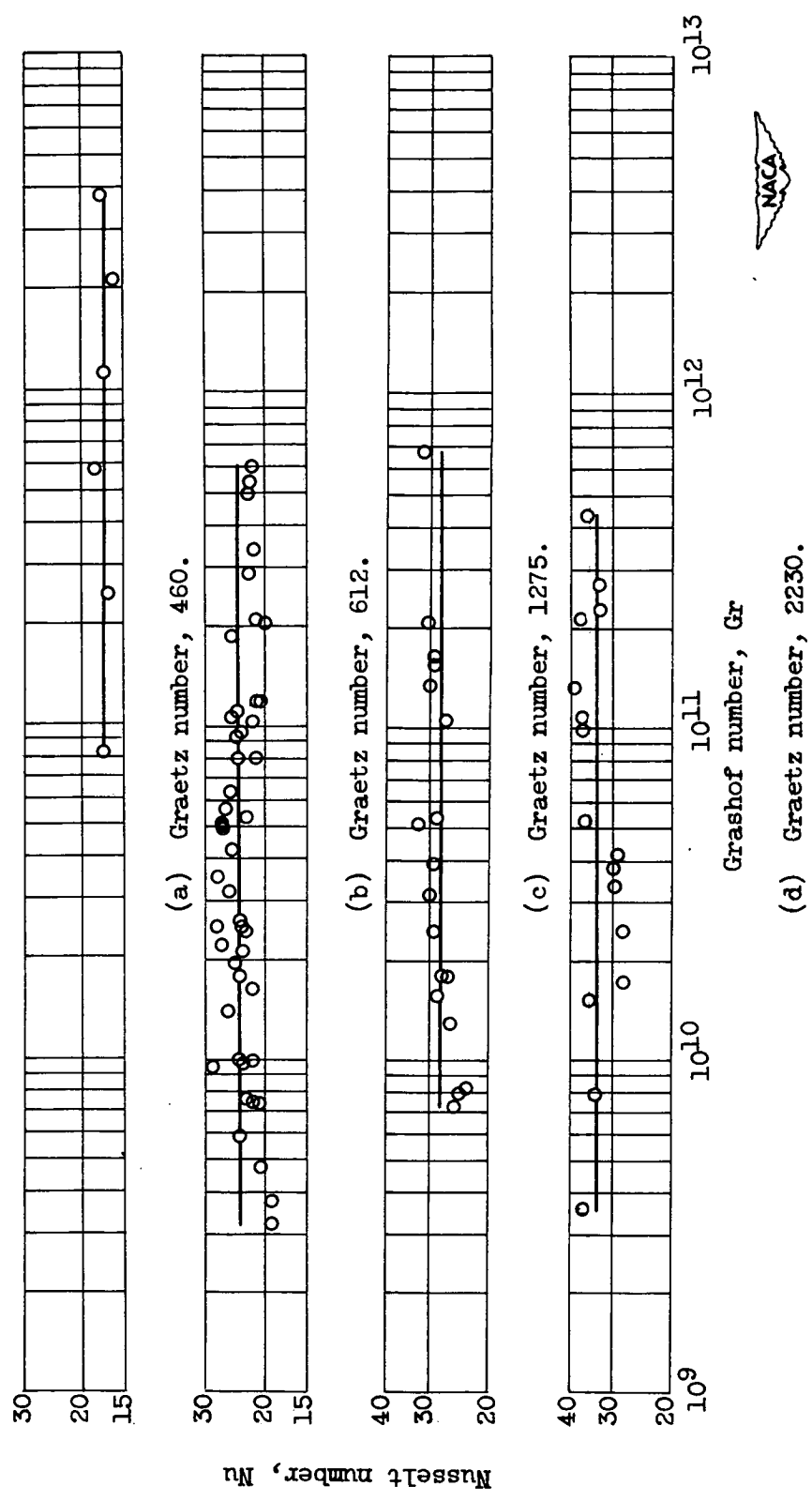


Figure 4. - Effect of Grashof number on blade-to-coolant heat-transfer data from forced-convection water-cooled turbine for several constant Graetz numbers. Fluid properties based on average coolant temperature.

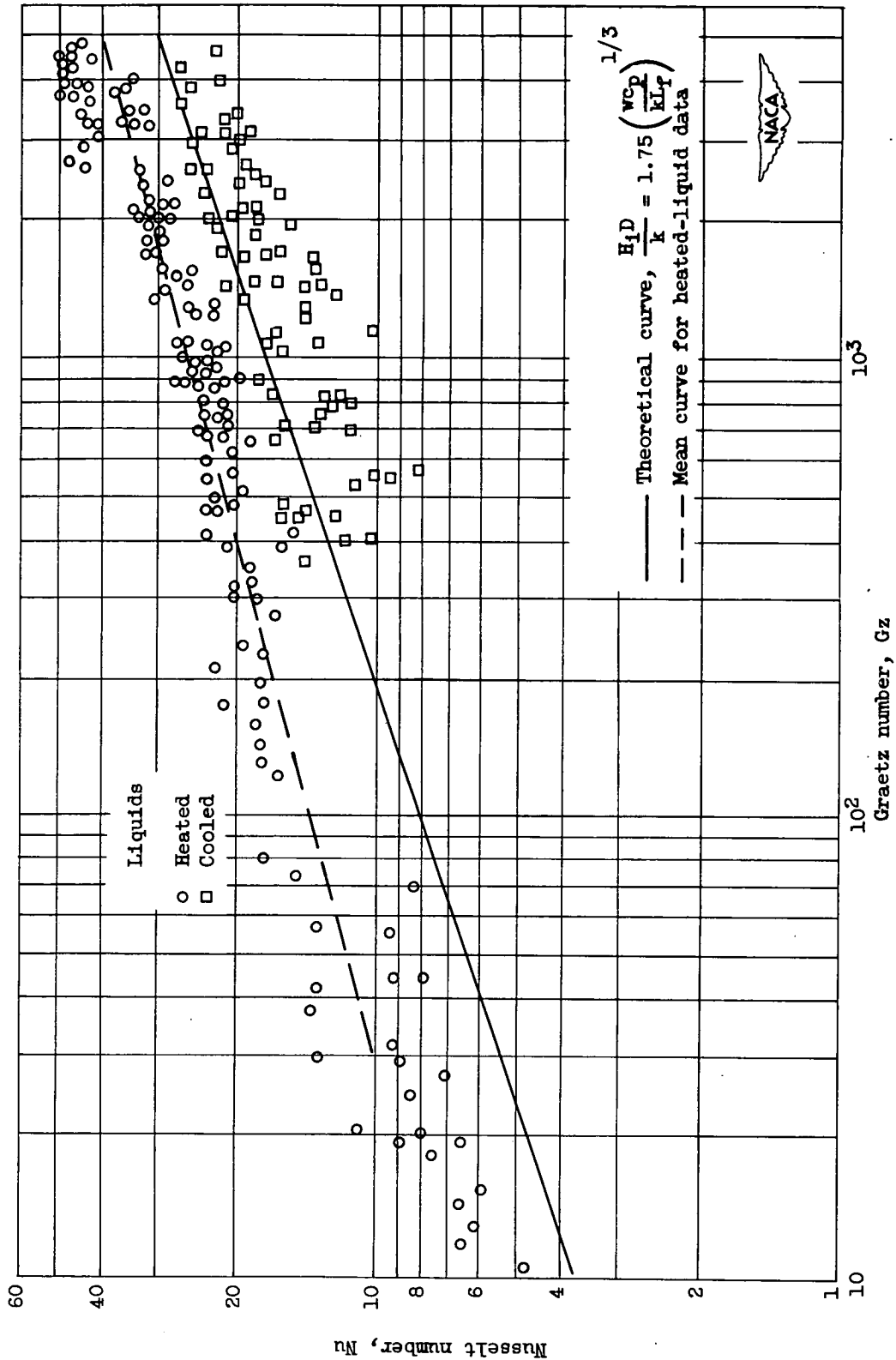


Figure 5. - Laminar-flow heat-transfer data (reference 1), theoretical correlation curve (reference 1), and mean-curve through data for heated liquids in stationary tubes.

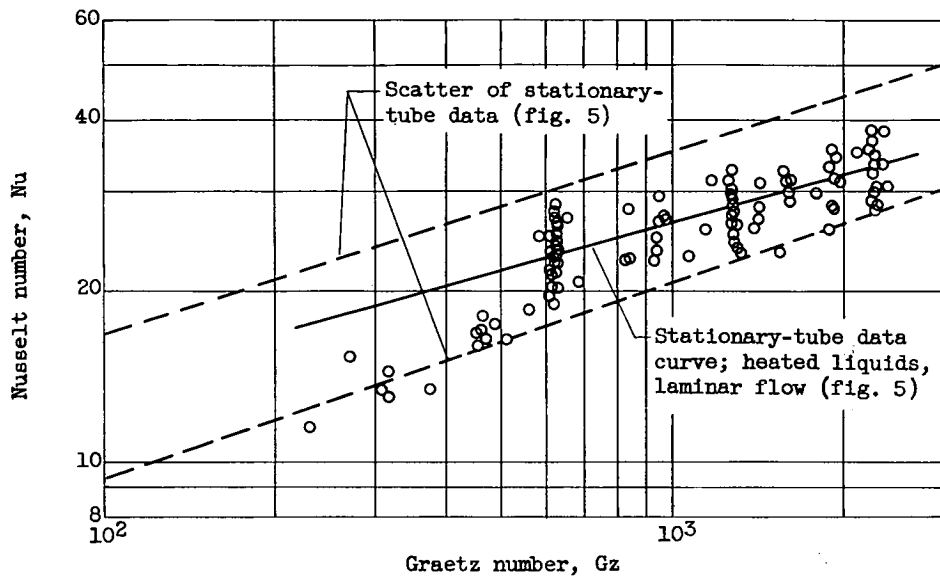


Figure 6. - Comparison of blade-to-coolant heat-transfer data from forced-convection water-cooled turbine with curve for stationary-tube data (fig. 5) for laminar flow of heated liquids. Fluid properties based on average coolant temperature.

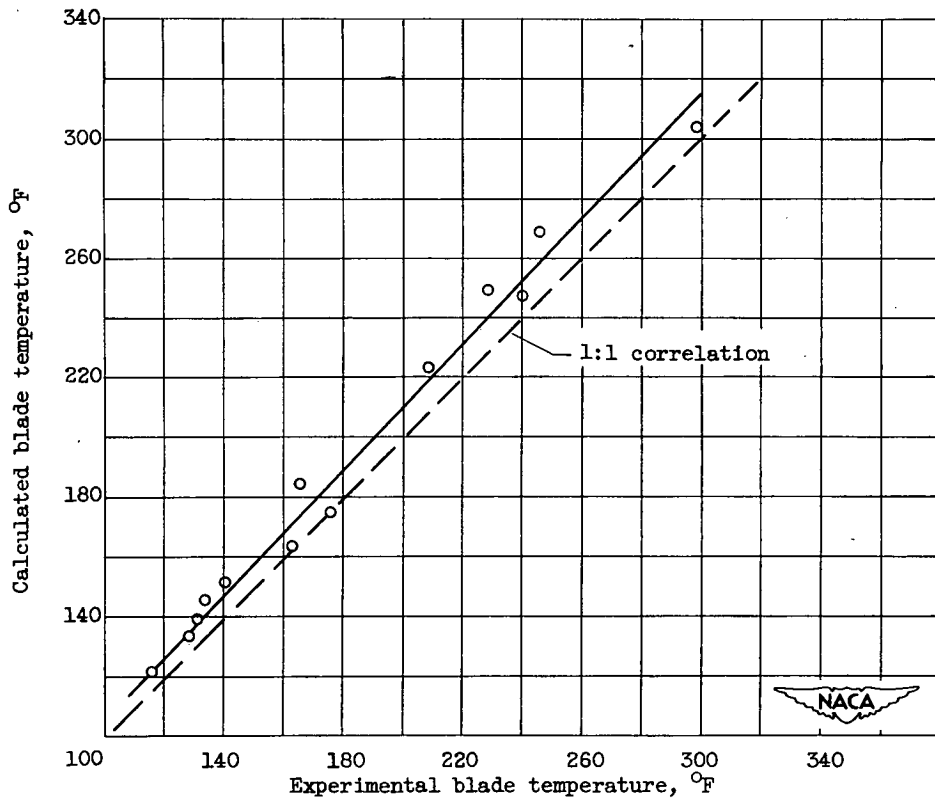


Figure 7. - Comparison of calculated average midspan blade temperatures with experimental data using blade-to-coolant coefficients from stationary-tube heated-liquid data and theoretical gas-to-blade coefficients.